## Design Principles and Calculations - Slab Design

## The Hambro Slab

The slab component of the Hambro D500 Composite Floor System behaves as a continuous one-way slab carrying loads transversely to the joists, and often is required to also act as a diaphragm carrying lateral loads to shear walls or other lateral load resisting elements.

At the present time, the Hambro slab has been designed by conventional ultimate strength design procedures of ACI 318 and section capacities are based on ultimate strength principles while the moments are still determined by using the elastic moment coefficients for continuous spans. This procedure is present in most other building codes.

In accordance with most building code requirements, the Hambro slab capacity is determined for two basic loading arrangements: a) uniform dead and live load extending in all directions, and b) a "standard" concentrated live load, applied anywhere, together with the slab dead loads. It is important to remember that the live load arrangements of $a$ ) and b) do not occur simultaneously. These loading arrangements will be discussed in detail.

## Moment:

The basic ultimate strength moment expressions from ACl are shown below:

$$
\begin{equation*}
M_{u}=A_{s} a_{u} d \tag{1}
\end{equation*}
$$

Where

$$
\begin{aligned}
& M_{u}=\underset{(f t .-k i p s / f i t . \text { width })}{\text { ultimate moment capacity of slab }} \\
& \text { (ft.-kips/fit.width) } \\
& A_{s}=\text { area of reinforcing mesh in the direction of } \\
& \text { the slab span (in. }{ }^{2 / f t} \text {. width) } \\
& a_{u}=\varnothing f_{y}(1-.59 w) / 12000 \\
& \emptyset=\text { flexure factor }=0.9 \\
& f_{y}=\text { yield strength of reinforcing mesh }=60,000 \mathrm{psi} \\
& \text { (or as calculated by the } \mathrm{ACl} \text { offset provision) } \\
& w=p \frac{f_{y}}{f^{\prime}{ }_{c}} \\
& p=\text { tension steel ratio }=A_{s} / b d \\
& f_{c}^{\prime}=\text { compressive strength of concrete } \\
& =3,000 \mathrm{psi} \\
& b=\text { unit slab width }=12 \text { inches } \\
& d=\text { distance from extreme compression fibre to } \\
& \text { centroid of reinforcing mesh (in.) } \\
& =1.6 \text { inches for } 2-1 / 2 \text { inch slab }
\end{aligned}
$$

It is a simple matter, then, to determine $M_{u}$ for any combination of $A_{s}$ and $d$. Taking into account $3 / 4$ inch concrete cover, " $d$ " is taken to be 1.6 inches for the $2-1 / 2$ inch slab thickness. The ACl Ultimate Strength Design Handbook Vol. 1, Publication SP17, contains
tabulated values for " $a_{u}$ " (note that $a_{u}$ increases as the tension steel ratio " $p$ " decreases).

## Crack Control Provisions:

When design yield strength $f_{y}$ for tension reinforcement exceeds 40,000 psi, cross sections of maximum positive and negative moment shall be so proportioned that the quantity $z$ given by:

$$
z=f_{s} \sqrt[3]{d_{c} A}
$$

(2) $[\mathrm{ACl}$ 10.6.3.4]
does not exceed 175 kips per in. for interior exposure and 145 kips per in. for exterior exposure. Calculated stress in reinforcement at service load $f_{s}$ (kips per sq. in.) shall be computed as the moment divided by the product of steel area and internal moment arm. In lieu of such computations, $f_{s}$ may be taken as 60 percent of specified yield strength $f_{y}$.


Fig. 1
Considering the negative moment region where the mesh rests directly on the embedded top chord connector, the centroid of the mesh is 1.6 inches above the extreme concrete compression fibre. With 6 inches wire spacing and $t=2-1 / 2$ inches, $d_{c}=0.9$ inches;
$A$ (hatched area) $=1.8 \times 6=10.8$ in. ${ }^{2}$;
Using $f_{s}=60 \%$ of $60 \mathrm{ksi}=36 \mathrm{ksi}$;
" $z$ " in formula 2 becomes 77 kips per inch.
Even for a 3 inch slab with the lever arm still at 1.6 inch and $d$ = 1.4 inch, " $z$ " = 103 kips per inch, well under the allowable.

## Shear Stress

The ultimate shear capacity, $v_{c u}$, which is a measure of diagonal tension, is unaffected by the embedment of the top chord section as this principal tensile crack would be inclined and radiate away from the $z$ section. Furthermore, there is no vertical weak plane through which a premature "punching shear" type of failure could occur.

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A check of the $v_{c u}$ capacity for $d=1.6$ inch $\left(2-1 \frac{1}{2}\right.$ inch slab) and $f^{\prime}{ }_{c}=3,000$ psi is shown:

$$
\begin{align*}
& v_{c u}=\frac{V_{u}}{\emptyset b d}  \tag{3}\\
& V_{u}=\emptyset v_{c u} b d \tag{4}
\end{align*}
$$

Substituting the following values in (4): $v_{c u}=2 \sqrt{f^{\prime}}{ }_{c}=100 \mathrm{psi}$, $b=12 ", d=1.6 ", \emptyset=.85$, results in:

$$
V_{u}=1,785 \mathrm{lbs} .
$$

With a Load Factor of 1.7 , the $2-1 / 2$ inch slab spanning 4 feet $1-1 / 4 \mathrm{inch} \mathrm{c} / \mathrm{c}$ has a safe shear capacity of 525 psf .

## Deflection:

The span / thickness ratio 49.5 / $2.5=20$ is less than the maximum allowable 24 for the one end continuous condition. Slab deflection, $\Delta$, due to a uniformly distributed load, can be written as:

$$
\begin{align*}
& \Delta=\frac{K_{I} w L^{4}}{E_{c} I_{c}}  \tag{5}\\
& \frac{\Delta}{L}=\frac{K_{I} w L^{3}}{E_{c} I_{c}} \tag{6}
\end{align*}
$$

For the same $w, E_{c}$ and slab end conditions, (6) can be rewritten:

$$
\frac{\Delta}{L}=K_{2} \frac{L^{3}}{I_{c}}
$$

Hence, the $\Delta / L$ ratios of different floors can be used to assess relative deflection.

Example:
Hambro 2- $1 / 2$ INCH SLAB / - 4 FOOT $1-1 / 4$ INCH SPAN

$$
I_{c}=12(2.5)^{3} / 12=15.6 \text { in. }{ }^{4}
$$

$$
\Delta / L=\mathrm{L}^{3} / I_{c}=(4.1)^{3} / 15.6=4.4
$$

## $7-1 / 2$ INCH SLAB / 20 FOOT SPAN

$$
I_{c}=12(7.5)^{3} / 12=422 \text { in. } .^{4}
$$

$$
\Delta / L=\mathrm{L}^{3} / l_{c}=(20)^{3} / 422=19
$$

Clearly, then, the Hambro slab deflections expressed in terms of $\Delta / L$ are less with Hambro than with a $7-1 / 2$ inch slab spanning 20 feet.

## Uniform Live Load Arrangement:

Generally, the slab capacity is checked against the condition where the dead and live loads are uniformly distributed.

A load factor of 1.7 is used for both dead and live load capacities. For a more exact analysis, factored loads of $1.4 D$ and 1.7L should be considered.

Refer to Fig. 2 for location of the design moments below.
Ultimate positive moment, $M_{u}$ : exterior span $1.7 w_{s} L_{l}^{2 / 11} \ldots \ldots . . . . . . . . . . . .$. location 1 interior span $1.7 w_{s} L_{2}{ }^{2} / 16$ $\qquad$ location 3

Ultimate negative moment, $M_{u}$ :
exterior span $1.7 w_{s} L^{2 / 10}$ $\qquad$ location 2
interior span $1.7 w_{s} L_{2}{ }^{2 / 11}$ $\qquad$ location 4

Where

$$
\begin{aligned}
L_{1}, L_{2}= & \begin{array}{c}
\text { clear span (ft.) is less than } 13 / 4 \text { " less } \\
\\
\\
\text { than joist spacing. }
\end{array} \\
L & =\text { average of } L_{1} \text { and } L_{2}(\mathrm{ft} .) \\
w_{s} & =\text { total design load (dead }+ \text { live) in psf }
\end{aligned}
$$

Note that a conservative load factor of 1.7 has been used for dead and live load.

When $L_{1}=L_{2}$, maximum mesh stress occurs at location 2. When $L_{1} \leq 0.9 L_{2}$, maximum mesh stress occurs at location 4. The slab load tables are reproduced in Table 1.

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Table 1 - Slab Capacity Chart (Total Load in psf)

| SLAB <br> THICKNESS (t) |  | $\underset{\text { (inch) }}{d}$ | MESH SIZE (all 6 in. $x$ 6 in.) F'c $=3,000$ psi $F_{y}=60,000 \mathrm{psi}$ | JOIST SPACING |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 4'-1 1/4" |  | 5'-0" |  |
|  |  | Exterior |  | Interior | Exterior | Interior |
| 2 | $\begin{aligned} & t \geq 2-3 / 4 \text { in. and } \\ & t \leq 3-5 / 8 \text { in. } \end{aligned}$ |  | 1.6 in . | $\begin{aligned} & 6 \times 6-2.9 / 2.9 \\ & 6 \times 6-4.0 / 4.0 \end{aligned}$ | $\begin{aligned} & 159 \\ & 212 \end{aligned}$ | $\begin{aligned} & 175 \\ & 233 \end{aligned}$ | $\begin{aligned} & 108 \\ & 143 \end{aligned}$ | $\begin{aligned} & 118 \\ & 157 \end{aligned}$ |
| No Chair | $\begin{aligned} & t \geq 3-5 / 8 \text { in. and } \\ & t \leq 5 \text { in. } \end{aligned}$ |  | 1.6 in . | $6 \times 6-4.0 / 4.0$ 2 layers $6 \times 6-2.1 / 2.1$ 2 layers $6 \times 6-2.9 / 2.9$ | $\begin{aligned} & 219 \\ & 226 \\ & 304 \end{aligned}$ | $\begin{aligned} & 241 \\ & 248 \\ & 334 \end{aligned}$ | $\begin{aligned} & 148 \\ & 152 \\ & 204 \end{aligned}$ | $\begin{aligned} & 162 \\ & 167 \\ & 223 \end{aligned}$ |
| $5$ | $\begin{aligned} & t \geq 3 \text { in. and } \\ & t \leq 3-5 / 8 \text { in. } \end{aligned}$ | 2.1 in. | $\begin{aligned} & 6 \times 6-2.9 / 2.9 \\ & 6 \times 6-4.0 / 4.0 \end{aligned}$ | $\begin{aligned} & 220 \\ & 295 \end{aligned}$ | $\begin{aligned} & 241 \\ & 324 \end{aligned}$ | $\begin{aligned} & 148 \\ & 198 \end{aligned}$ | $\begin{aligned} & 162 \\ & 217 \end{aligned}$ |
| 1/2" Rod Shop Welded to Top Chord | $\begin{aligned} & t \geq 3-5 / 8 \text { in. and } \\ & t \leq 5 \text { in. } \end{aligned}$ | 2.1 in. | $6 \times 6-4.0 / 4.0$ 2 layers $6 \times 6-2.1 / 2.1$ 2 layers $6 \times 6-2.9 / 2.9$ | $\begin{aligned} & 296 \\ & 307 \\ & 415 \end{aligned}$ | $\begin{aligned} & 326 \\ & 336 \\ & 457 \\ & \hline \end{aligned}$ | $\begin{aligned} & 200 \\ & 206 \\ & 279 \end{aligned}$ | $\begin{aligned} & 219 \\ & 226 \\ & 306 \end{aligned}$ |
| Chair | $\begin{aligned} & t \geq 3-5 / 8 \text { in. and } \\ & t \leq 5 \text { in. } \end{aligned}$ | 2.6 in. | $\begin{array}{r} 6 \times 6-4.0 / 4.0 \\ 2 \text { layers } 6 \times 6-2.1 / 2.1 \\ 2 \text { layers } 6 \times 6-2.9 / 2.9 \end{array}$ | $\begin{aligned} & 353 \\ & 363 \\ & 497 \end{aligned}$ | $\begin{aligned} & 387 \\ & 399 \\ & 545 \end{aligned}$ | $\begin{aligned} & 236 \\ & 244 \\ & 333 \end{aligned}$ | $\begin{aligned} & 259 \\ & 267 \\ & 365 \end{aligned}$ |

Note: Slab capacities are based on mesh over joists raised as indicated.


Fig. 2

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## Concentrated Live Load Requirements

Building Codes usually stipulate designing to possible concentration of live loads. Typical examples of some of these situations are listed in Table 2. Check your local codes for exact requirements.

The loads are applied over an area $2-1 / 2$ feet by $2-1 / 2$ feet and it is important to remember that they are not applied simultaneously with the uniformly distributed live loads.

## Table 2

| USE | MINIMUM |
| :--- | :---: |
| CONCENTRATED LOAD (Ib.) |  |
| Classrooms | 1000 |
| Floors of offices, manufacturing buildings, hospital wards, stages | 2000 |
| Floor areas used by passenger cars | $2500^{*}$ |

* Some building codes use 2000 lbs.


Section A-A

Fig. 3
Lateral Distribution of Concentrated Loads

The intensity of concentrated loads on slabs is reduced due to lateral distribution. One of the accepted methods of calculating the "effective slab width," which is used by Hambro, actually appears in Section 317 of the British Standard Code of Practice CP114 and is reproduced in Fig. 3. Note that the amount of lateral distribution increases as the load moves closer to mid
span, and reaches a maximum of 0.3 L to each side; the effective slab width resisting the load is a maximum of load width $+0.6 L$.

An abbreviated summary of the calculations is shown in Tables 3 and 4.

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TABLE 3 -Concentrated Loads with 4 Foot-1-1/4 inch Joist Spacing

| CONCENTRATED LOAD | $\begin{gathered} \text { SLAB } \\ \text { THICKNESS } \end{gathered}$ | $\mathrm{MESH}$ SIZE | SPECIAL REMARKS |  |
| :---: | :---: | :---: | :---: | :---: |
| 2000 lbs. on 2 feet-6 inch square area (office building) | 2-1/2 in. | 6×6-W2.9 | Extra layer @ (1) | $\begin{gathered} \text { No } \\ { }_{2}^{\text {chairs" on }} \end{gathered}$ |
|  |  | 6x6-W2.9 | Single layer throughout but $\mathrm{S}_{1}=3$ feet-10 inch max. |  |
|  | 3 in. | $6 \times 6-\mathrm{W} 2.9$ | Extra layer @ (1) and (2) |  |
|  |  | 6 $\times 6$ - W2.9 | Single layer throughout but $S_{1}=4$ feet max. |  |
|  |  | $6 \times 6-\mathrm{W} 2.9$ | Single layer throughout |  |
| 2500 lbs. on 2 feet-6 inch square area plus 2 inch asphalt wearing surface | 3 in. | $6 \times 6-\mathrm{W} 2.9$ | Extra layer @ (1) and (2) |  |
|  |  | $6 \times 6-\mathrm{W} 2.9$ | Single layer throughout but $S_{1}=2$ feet-10 inch max. |  |
| 4000 lbs. on 3 feet-6 inch square area (office building for some codes) | 2-1/2 in. | $6 \times 6-\mathrm{W} 4.0$ | $\mathrm{S}_{1}=4$ feet | $\begin{gathered} \text { No } \\ \text { "chairs" on } \\ 2 \end{gathered}$ |
|  | 3 in. | 6x6-W2.9 | Extra layer @ (1) and (2) |  |
|  |  | 6x6-W2.9 | Single layer throughout but $S_{1}=2$ feet-10 inch max. |  |

*Some building codes use different bearing areas.
TABLE 4 -Concentrated Loads with 5 feet-1- $1 / 4$ inch Joist

| Spacing |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| CONCENTRATED <br> LOAD | SLAB <br> THICKNESS | MESH <br> SIZE | SPECIAL <br> REMARKS |  |
| 2000 lbs. on2 feet- 6 <br> inch square area <br> (office building) | 3 in. | $6 \times 6-\mathrm{W} 2.9$ | Extra layer @ (1) and (2) | "chairs" on |
| 4000 Ibs. on 3 feet- 6 <br> inch square area <br> (office for some codes) | 3 in. | $6 \times 6-\mathrm{W} 4.0$ | Extra layer @ (1) and (2) | "chairs" on |

## Concrete Mix

Top size of the coarse aggregate should not exceed $3 / 4$ inch or as dictated by applicable codes. A slump of 4 inches is recommended.


